JOURNAL OF BALTIC SCIENCE EDUCATION

ISSN 1648-3898 /Print/
ISSN 2538-7138 /Online/

This is an open access article under the Creative Commons Attribution 4.0 International License

# COGNITIVE DIAGNOSIS OF UPPER-SECONDARY STUDENTS' CONCEPTIONS IN IONIZATION EQUILIBRIUM BASED ON THE RULE SPACE MODEL

**Limin Wang,**
**Qing Zhou,**
**Le Ji,**
**Huiying Zhang,**
**Ye Li**

**Abstract.** *Evaluating students' cognitive understanding of core concepts and identifying learning path aligned with their cognitive developmental patterns are critical for enhancing conceptual mastery in chemistry. This study employed the Rule Space Model to investigate students' cognitive characteristics and learning path in comprehending the conceptual network centered on "ionization equilibrium." Within this framework, nine key attributes were identified, and corresponding assessment instruments comprising 23 test items were developed. The assessment was administered to 340 upper-secondary school students in China, with systematic scoring and analysis of their responses. The findings reveal that: (1) The assessment demonstrates satisfactory reliability and validity; (2) Students exhibited significant cognitive challenges in four attributes: "Ionization degree," "Ionization equilibrium constant," "Solution acidity/alkalinity and pH," and "Acid-base titration". (3) The predominant learning path for students to construct a conceptual network is: Electrolyte → Strong /weak electrolyte → Ionization equilibrium → Ionization degree → Ionic product of water → The shift in ionization equilibrium → Ionization equilibrium constant → Solution acidity/alkalinity and pH → Acid-base titration.*

**Keywords:** *ionization equilibrium, rule space model, cognitive diagnostic assessment, cognitive structure, learning path*

**Limin Wang, Qing Zhou**
*Shaanxi Normal University, P. R. China*
**Le Ji**
*Xi' an Tieyi High School, P. R. China*
**Huiying Zhang**
*Hanshan Normal University, P. R. China*
**Ye Li**
*Shaanxi People's Education Press, P. R. China*

## Introduction

In the 1990s, the Zero Point Project — a collaboration between the Spencer Fund and Harvard Graduate School of Education — developed and implemented the "Teaching for Understanding" framework. This initiative has catalyzed a paradigm shift in international science educational research, with growing emphasis on two interrelated goals: (1) systematically diagnosing students' conceptual understanding, and (2) developing pedagogical strategies to advance it (Gaigher et al., 2007; Jin & Wong, 2023; Milligan & Wood, 2010; Perkins, 1993). The framework's theoretical foundation rests on a critical distinction: conceptual understanding constitutes a connected knowledge structure, fundamentally different from isolated factual recall. Bransford et al. (2000) posit that such understanding emerges only when learners transform fragmented information into coherent cognitive schemata through deliberate conceptual integration. Empirical observations confirm this process — students initially operate with disconnected knowledge elements, progressively achieving structural coherence through iterative cycles of assimilation and reorganization (Alao & Guthrie, 1999; Bransford et al., 2000). This theoretical clarity creates a corresponding dual imperative for science educational research. First, accurate assessment requires diagnostic tools capable of mapping the structural properties of students' conceptual networks. Second, pedagogical innovation demands the identification of domain — specific construction trajectories — sequenced developmental pathways that align with learners' evolving cognitive architectures. (Anderson & Schönborn, 2008; Stevens et al., 2010).

The ionization equilibrium is a fundamental topic in upper-secondary school chemistry. This topic typically encompasses contents such as electrolytes, strong electrolytes, weak electrolytes, ionization equilibrium, pH, acid-base titration (Aydin-Gunbatar & Akin, 2022; Ministry of Education of the People's Republic of China, 2017). These concepts form an interrelated conceptual network. A comprehensive understanding of this topic enables



https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

students to interpret real-world phenomena, such as the acid-base balance in the human body and the neutralization of gastric acid by antacids (He et al., 2022). Additionally, this content serves as a foundation for advanced studies in acid-base chemistry and electrochemistry (Kousathana et al., 2005; Nakiboglu et al., 2024). However, existing studies found that students often do not fully understand those concepts (Artdej et al., 2010; Lu & Bi, 2016). Furthermore, it is also a challenge for students to connect them to understand complex phenomena and solve real-world problems, which in turn leads to difficulties in learning subsequent, more advanced topics (Jiménez-Liso et al., 2020).

Existing approaches to evaluate students' conceptual understanding include using two-tier and three-tier diagnostic tests to diagnose students' alternative conceptions (Cheong et al., 2014; McClary & Bretz, 2012; Özmen, 2008; Yeo et al., 2022), as well as using the Rasch model to evaluate students' conceptual understanding level (Hadenfeldt et al., 2013; Planinić et al., 2019; Yang et al., 2024). While two-tier and three-tier diagnostic instruments effectively identify specific misconceptions and their underlying causes (Mintzes et al., 2005; Treagust, 1988), they lack the capacity to reveal comprehensive cognitive structures. On the other hand, the Rasch model quantifies students' conceptual proficiency levels but fails to delineate granular learning pathways for conceptual system construction (Chen et al., 2016).

Cognitive Diagnostic Assessment (CDA) offers enhanced diagnostic capabilities by evaluating learners' knowledge structures and processing skills (Leighton & Gierl, 2007). This methodology employs diagnostic assessments combined with psychometric models from cognitive psychology to analyze response patterns (Zhan et al., 2021). Through this approach, CDA identifies the cognitive processes, knowledge components, and strategic competencies mobilized during task completion, enabling systematic diagnosis of students' cognitive structures and the acquisition of their knowledge states, which allows for the inference of learning paths based on inclusion relationships between these states (Wu et al., 2021). The diagnostic framework relies on cognitive diagnosis models (CDMs) — psychometric models that operationalize relationships between students' item response patterns and their latent knowledge states (Xin et al., 2022).

Among cognitive diagnostic models (CDMs), the Rule Space Model (RSM) has demonstrated robust diagnostic validity through extensive empirical validation (Chen et al., 2017; Gao et al., 2022; Lin et al., 2022; Tatsuoka, 1983). This study employs RSM to diagnose cognitive characteristics at both group and individual student levels, and explore the cognitive paths involved in upper-secondary students' construction of the conceptual networks of ionization equilibrium. The approach of this study provides research insights for science education researchers to explore the latent cognitive processes of students' conceptual understanding.

### *Rule Space Model*

The Rule Space Model (RSM) is a probabilistic framework integrating item response theory (IRT) and pattern classification techniques to diagnose latent cognitive attributes (Tatsuoka, 1995). Its operationalization involves six axiomatic phases:

**Attribute Operationalization:** Knowledge components, skills, and strategic competencies within the assessment domain are formally defined as discrete attributes. These attributes are systematically combined according to their ontological dependencies to generate all theoretically possible cognitive configurations, termed attribute mastery patterns (AMPs).

**Ideal Response Generation:** Under idealized conditions (assuming zero stochastic guessing and measurement error), each AMP is mapped to its corresponding deterministic response vector, designated as the ideal response pattern (IRP).

**Psychometric Calibration:** Through IRP analysis, a latent ability metric ($\theta$) is parametrically linked to each AMP. This enables computation of the caution index $\zeta$ - a Mahalanobis-based metric quantifying the divergence between observed response patterns and $\theta$-level expectations.

**Reference Point Establishment:** A set of canonical rule points ($\theta$, $\zeta$) is thereby established, with each point uniquely representing an AMP in the rule space.

**Examinee Profiling:** Actual test responses are analyzed to estimate individual ability levels ($\theta_i$) and caution indices ($\zeta_i$), generating classification coordinates ($\theta_i$, $\zeta_i$) for each examinee.

**Cognitive Classification:** Using Mahalanobis distance minimization (critical $\chi^2$ threshold = 5.99, $\alpha$=.05), each ($\theta_i$, $\zeta_i$) coordinate is assigned to the nearest canonical rule point, thereby diagnostically mapping examinees to specific AMPs that characterize their cognitive architectures.


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

*Research Problem*

The literature indicates that it is difficult for students to integrate concepts in the topic of ionization equilibrium into a connected knowledge structure and to develop a coherent conceptual understanding. However, the existing studies only investigated specific concepts within this topic (e.g., electrolytes, ionization equilibrium, pH), while failing to provide a systematic diagnosis of students' cognitive structures or identify optimal instructional pathways for constructing holistic conceptual frameworks.

*Research Aim and Research Questions*

The purpose of this study was to utilize RSM to assess students' conceptual understanding of *ionization equilibrium* and to explore their cognitive structures and cognitive paths. The study aimed to address the following research questions:

1. What are the cognitive strengths and weaknesses of upper-secondary students' conceptual understanding of *ionization equilibrium*?
2. What are the cognitive characteristics of upper-secondary students' proficiency in *ionization equilibrium*?
3. What are the learning pathways of upper-secondary students' conceptions in *ionization equilibrium*?

## Research Methodology

*General Background*

This study employed RSM — a cognitive diagnostic model — to analyze Chinese students' cognitive structures. To collect data, an instrument based on cognitive diagnostic theory was developed and used to assess Grade 11 Chinese students' understanding of the "Ionization Equilibrium". Before implementing the test, students had all completed the learning of "Ionization Equilibrium" as presented in the chemistry curriculum.

*Participants*

Participants were Grade 11 students from Shaanxi Province, a district in the northwest part of mainland China. Given the constraints of funding and resources, a convenience sampling method was employed to select participants. An ordinary upper-secondary school was purposefully selected, and its students came from middle-class families in urban areas, having a wide range of academic achievements. A total of 340 Grade 11 students from six classes at this school were included in the final sample.

*Instrument and Procedures*

**1) Identify cognitive attributes and the hierarchy structure of attributes**

Cognitive attribute refers to the necessary knowledge, skills or strategies for students to complete the test items. In this study, the cognitive attributes of ionization equilibrium were identified by the Chinese National Curriculum Standards of Chemistry for Upper-Secondary School and three current editions of upper-secondary school chemistry textbooks in China (Ministry of Education of the People's Republic of China, 2017).

The curriculum standards define the curriculum objectives in both the required sessions and elective sessions as follows: the students should (1) understand the concept of electrolytes; (2) distinguish between strong and weak electrolytes; (3) comprehend ionization equilibrium in weak electrolyte aqueous solutions; (4) interpret the significance of ionization equilibrium constants; (5) analyze water ionization and its ion product constant; (6) evaluate solution acidity, alkalinity and pH; (7) demonstrate proficiency in measuring solution pH. Based on these standards, seven cognitive attributes: "Electrolyte," "Strong/weak electrolyte," "Ionization equilibrium," "The shifting of ionic equilibrium," "Ionization equilibrium constant," "Ionic Product of Water," and "Solution acidity/alkalinity and pH" could be extracted.

Subsequently, a comparative analysis of knowledge structures and instructional sequences across three current editions of upper-secondary school chemistry textbooks in China was conducted (denoted as Editions I, II, and III; see Table 1).


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

**Table 1**
*Sequence of Knowledge Arrangement in Textbooks*

| Editions of textbooks | Sequence of knowledge arrangement |
|---|---|
| I | Electrolyte→Strong/weak electrolyte→Ionization equilibrium→The shifting of ionic equilibrium→Ionization equilibrium constant→Ionic Product of Water→Solution acidity/alkalinity and pH→Acid-base titration |
| II | Electrolyte→Strong/weak electrolyte→Ionic Product of Water→Ionization equilibrium→Solution acidity/alkalinity and pH→Ionization equilibrium constant→Ionization degree→The shifting of ionic equilibrium→Acid-base titration |
| III | Electrolyte→Strong/weak electrolyte→Ionization equilibrium→Ionization equilibrium constant→Ionization degree→The shifting of ionic equilibrium→Ionic Product of Water→Solution acidity/alkalinity and pH→Acid-base titration |

The analysis of textbooks revealed that, in addition to the above seven attributes, the textbooks also included two further attributes: "Ionization degree" and "Acid-base titration". To sum up, this study ultimately identified seven attributes of "Ionization equilibrium". The detailed descriptions of attributes see Table 2.

**Table 2**
*Cognitive Attributes and Their Descriptions*

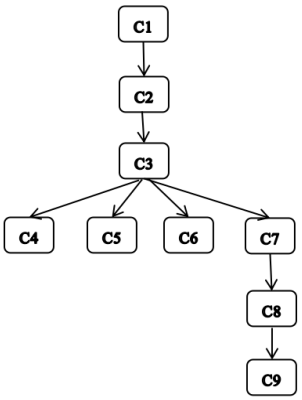
| Code | Attribute | Description |
|---|---|---|
| C1 | Electrolyte | Students can understand the definition of electrolytes and can identify whether a substance is an electrolyte or a non-electrolyte. |
| C2 | Strong/weak electrolyte | Students can distinguish strong electrolytes from weak electrolytes, understand the ionization process of strong and weak electrolytes in water, and analyze the particles and their conductivity in strong and weak electrolyte solutions. |
| C3 | Ionization equilibrium | Students can understand the ionization equilibrium of weak electrolytes in aqueous solutions and use equations to express the ionization of weak electrolytes. |
| C4 | The shift in ionization equilibrium | Students know the factors that affect the ionization equilibrium and how each factor affects the ionization equilibrium. |
| C5 | Ionization degree | Students can understand the definition of ionization degree and master the calculation formula, and they can also know the factors that affect the ionization degree and how each factor affects it. |
| C6 | Ionization equilibrium constant | Students can understand the definition of the ionization equilibrium constant and can use formulas to calculate it. They can also know the factors that affect the ionization equilibrium constant. |
| C7 | Ionic product of water | Students can understand the definition of the Ionic product of water (Kw) and can make calculations related to it. |
| C8 | Solution acidity/alkalinity and pH | Students can determine whether the solution is acidic or alkaline and calculate the pH of the solution. |
| C9 | Acid-base titration | Students can understand the principle of acid-base titration. They know the experimental operation of acid-base titration. They can analyze titration curve. |

The hierarchical structure between these attributes could be identified based on the descriptions of nine attributes and the order of knowledge in three textbooks, as shown in Figure 1.


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

**Figure 1**
*The Hierarchy Structure of Attributes*




"Electrolytes" (C1) was the foundational cognitive attribute in this topic. Building upon this concept, students could classify electrolytes into "Strong/weak electrolyte" (C2) based on their degree of ionization. On this basis, students could further understand the "Ionization equilibrium" (C3). Since students' mastery of four subsequent attributes—"The shift in ionization equilibrium" (C4), "Ionization degree" (C5), "Ionization equilibrium constant" (C6), and "Ionic Product of Water" (C7)— all depended on their grasp of "ionization equilibrium" (C3), these four attributes were subordinate to "Ionization equilibrium" (C3) and existed at the same hierarchical level.

The comprehension of "Solution acidity/alkalinity and pH" (C8) required prior knowledge of the relative concentrations of H+ and OH–, making the "Ionic product of water" (C7) a prerequisite for understanding "Solution acidity/alkalinity and pH" (C8). Finally, mastery of "Acid-base titration" (C9) relied on knowledge of "Solution acidity/alkalinity" (C8). Consequently, "Solution acidity/alkalinity and pH" (C8) served as the superordinate attribute for "Acid-base titration" (C9).

**2) Item design**

Two experienced science educators, and a PhD candidate in science education developed test items. The items were designed under dual validation criteria: (a) each attribute was examined at least three times; (b) congruence between inter-attribute relationships in test items and the predefined attribute hierarchy was ensured, to reduce random error and improve test reliability. To suppress random guessing behaviors, the assessment combines multiple-select items (partial-credit format) and constructed-response fill-in-the-blank items. A chemistry educational practitioner with approximately 15 years of experience in upper-secondary schools and a chemistry education professor and researcher who was familiar with chemistry and chemistry education reviewed the test questions to ensure alignment between content and measured attributes. Ambiguous questions were revised through discussion to strengthen content validity. Finally, 23 items (13 multiple-select questions and 10 fill-in-the-blank items) were retained to constitute the cognitive diagnostic instrument. The corresponding Q-matrix (Table 3) delineated item-attribute associations through binary coding, where "1" denotes attribute measurement and "0" its absence.

**Table 3**
*Q Matrix*

| Item | Attribute | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | C1 | C2 | C3 | C5 | C6 | C7 | C8 | C9 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 4 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

| Item | Attribute | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | C1 | C2 | C3 | C5 | C6 | C7 | C8 | C9 |
| 5 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 6 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 8 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 9 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 10 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 11 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 12 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 13 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 14 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 16 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 17 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 18 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 19 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 20 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 21 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 22 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 23 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

*Data Collection*

The test lasted 90 minutes, which was sufficient for all students to complete the assessment. After removing 22 incomplete datasets, the final analytic sample of 318 students' responses to the assessment test was obtained. 45.6% ($N = 145$) of the final sample were female students, whereas 54.4% ($N = 173$) were male. Before the test was implemented, researchers explained the purpose of this study to the students and clarified that the data collected would be used solely for the study. Their personal information would be kept confidential. The implementation of the test and the subsequent research obtained the informed consent of each student.

*Data Analysis*

**1) Instrument functioning**

The responses from the 318 participants were encoded as ID 1 through ID 318. A 0-1 scoring system was used for evaluating the students' answers, with 1 point awarded for correct answers and 0 points for wrong answers. The data was then input into Excel.

Data analysis consisted of two parts. Firstly, the data collected were analyzed using SPSS 26.0 to evaluate the reliability and construct validity of the testing instrument. Cronbach's alpha was used for assessing reliability (Field, 2009), which indicated the internal consistency of test items. As for the construct validity, a linear regression analysis was used to see if an item difficulty level could be predicted by the attribute content of the item.

**2) RSM analysis**

The collected data were analyzed using ConQuest 4.0 and MATLAB to obtain the attribute mastery patterns and attribute mastery probability of students. The specific analysis process was as follows:


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

ConQuest 4.0 was used to analyze the ability value ($\theta$) corresponding to each attribute mastery pattern. Based on this, the warning value ($\zeta$) — which quantified the deviation between a student's actual test item response pattern, and the expected response pattern at their ability level — was calculated. These two parameters jointly defined the "pure regular points ($\theta$, $\zeta$)" for each attribute mastery pattern. Next, ConQuest 4.0 estimated the actual response data of students, computed their ability values ($\theta_i$) and warning values ($\zeta_i$), and thereby generated individualized points to be classified ($\theta_i$, $\zeta_i$) for each student. By applying the Mahalanobis distance discriminant method in MATLAB, each student's point ($\theta_i$, $\zeta_i$) was assigned to the nearest pure regular point ($\theta$, $\zeta$) with the minimum Mahalanobis distance, provided that the distance was less than 5.99 (the critical value of $\chi^2$ at $\alpha = 0.05$). Through this classification, students' attribute mastery patterns, and their probabilities were derived, ultimately achieving the diagnosis of their cognitive structures.

## Research Results

### *Test Quality Analysis*

**1) Test reliability**

The 23 items' internal consistency, as measured by Cronbach's alpha coefficient, was .83, which indicated that the test was reliable.

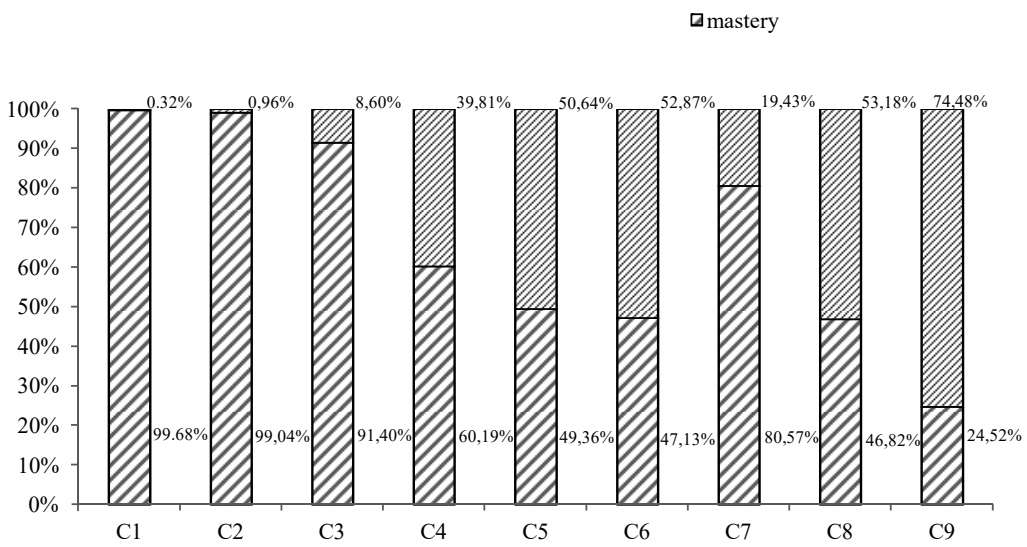
**2) Test validity**

Regarding the evaluation of validity, Dogan and Tatsuoka (2008) pointed out that in cognitive diagnosis tests, the more attributes an item examines, the more difficult it is. Therefore, it is possible to use the number of attributes measured by each item to predict item difficulty as a method to test the construct validity of the test. I took the number of cognitive attributes as the independent variable and the item difficulty as the dependent variable, and conducted a linear regression analysis. The adjusted $R^2$ value was .973, which indicated that the entire set of attributes accounted for 97.3% of the variance in item difficulty levels.

### *Attribute Mastery Probability*

The RSM analysis revealed the following descending order of attribute mastery probabilities: "Electrolyte" (C1, 99.68%) > "Strong /weak electrolyte" (C2, 99.04%) > "Ionization equilibrium" (C3, 91.40%) > "Ionic Product of Water" (C7, 80.57%) > "The shift in ionization equilibrium" (C4, 60.19%) > "Ionization degree" (C5, 49.36%) > "Ionization equilibrium constant" (C6, 47.13%) > "Solution acidity/alkalinity and pH" (C8, 46.82%) > "Acid-base titration" (C9, 24.52%).

**Figure 2**
*The Mastery and Non-mastery Probability of the Nine Attributes*





https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

Figure 2 shows that the students performed best in the attributes of C1 (Electrolyte), C2 (Strong/weak electrolyte), and C3 (Ionization equilibrium), with mastery probabilities exceeding 90%. Their performance in C7 (Ionic Product of Water) and C4 (The shift in ionization equilibrium) was better, with mastery probabilities above 50%. However, their performance in C5 (Ionization degree), C6 (Ionization equilibrium constant), and C8 (Solution acidity/alkalinity and pH) was not ideal. Notably, their performance in C9 (Acid-base titration) was poor, with a mastery probability of only 24%.

*Attribute Mastery Patterns*

In terms of attribute mastery patterns (AMPs), of the 318 students, 314 were successfully classified into 27 attribute mastery patterns (see Table 4). The classification rate reached 98.74%, indicating that the results of the diagnosis were reliable.

**Table 4**
*The Attribute Mastery Patterns*

| Code | AMP | θ | N | % |
|---|---|---|---|---|
| 1 | 000000000 | -3.78 | 1 | 0.31 |
| 2 | 100000000 | -3.34 | 2 | 0.63 |
| 3 | 110000000 | -2.87 | 24 | 7.55 |
| 4 | 111000000 | -2.44 | 9 | 2.83 |
| 5 | 111100000 | -2.09 | 4 | 1.26 |
| 6 | 111010000 | -2.09 | 6 | 1.89 |
| 7 | 111000100 | -2.089 | 2 | 0.63 |
| 8 | 111110000 | -1.48 | 4 | 1.26 |
| 9 | 111100100 | -1.48 | 11 | 3.46 |
| 10 | 111011000 | -1.48 | 7 | 2.20 |
| 11 | 111010100 | -1.78 | 14 | 4.40 |
| 12 | 111001100 | -1.48 | 6 | 1.89 |
| 13 | 111000111 | -1.78 | 5 | 1.57 |
| 14 | 111111000 | -0.64 | 4 | 1.26 |
| 15 | 111110100 | -0.90 | 35 | 11.01 |
| 16 | 111101100 | -0.64 | 10 | 3.14 |
| 17 | 111100110 | -0.90 | 18 | 5.66 |
| 18 | 111011100 | -1.18 | 12 | 3.77 |
| 19 | 111111100 | 0.61 | 16 | 5.03 |
| 20 | 111101110 | 0.36 | 17 | 5.35 |
| 21 | 111100111 | -0.64 | 18 | 5.66 |
| 22 | 111011110 | -0.64 | 31 | 9.75 |
| 23 | 111111110 | 2.38 | 4 | 1.26 |
| 24 | 111110111 | 0.11 | 13 | 4.09 |
| 25 | 111101111 | 0.87 | 32 | 10.06 |
| 26 | 111011111 | -0.14 | 6 | 1.89 |
| 27 | 111111111 | 3.57 | 3 | 0.94 |

As presented in Table 4, AMP15 (111110100) had the highest class probability. Approximately 11.01% of the students could distinguish between electrolytes and non-electrolytes, differentiate strong and weak electrolytes based on definitions, and analyze the ionization equilibrium of weak electrolytes, the ionic product of water, and


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

the degree of ionization. However, they did not master the ionization equilibrium constant, could not determine solution acidity/alkalinity, calculate pH values, or analyze acid-base titration.

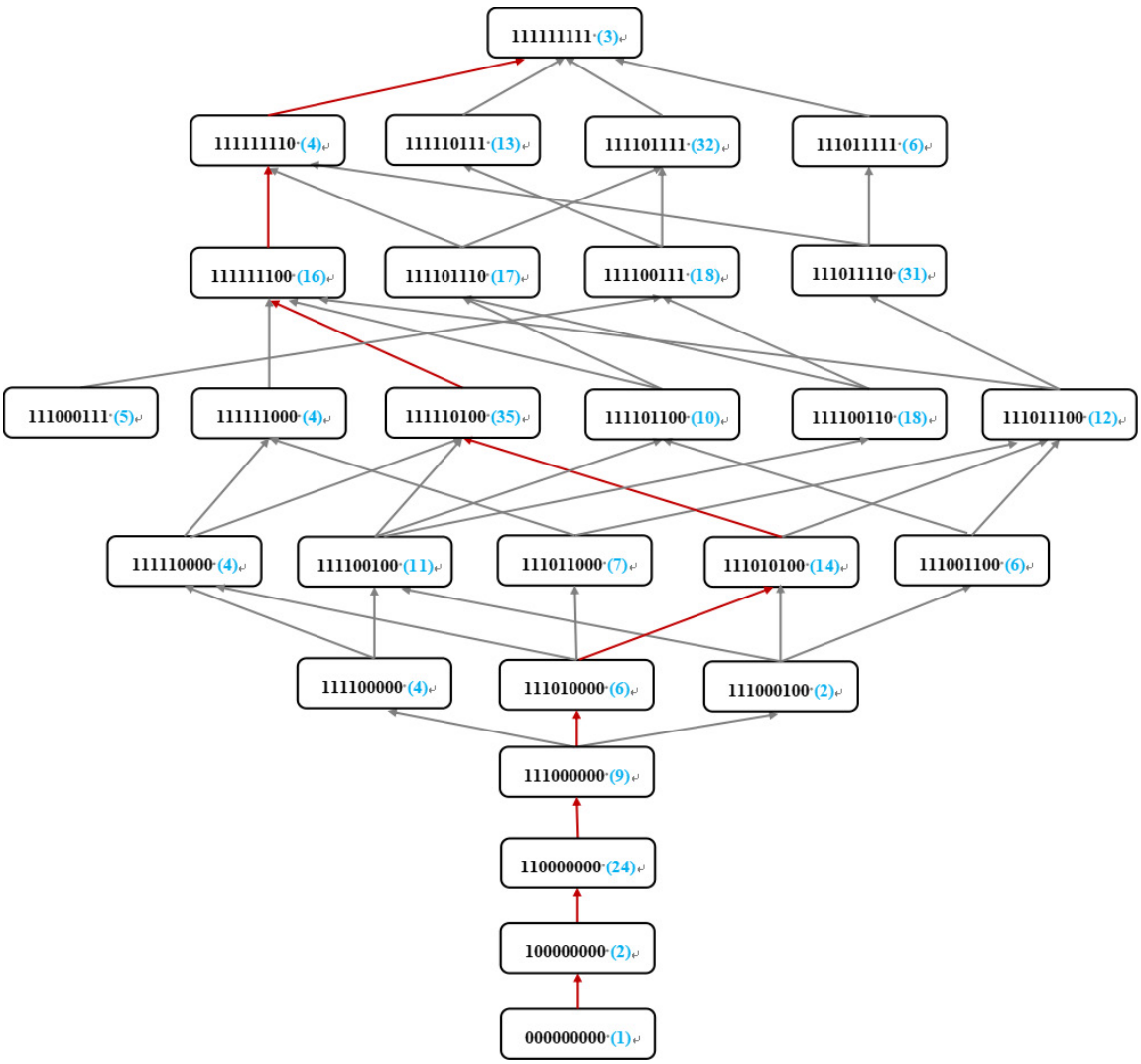
The second most prevalent latent class was AMP25 (111101111), to which 10.06% of the students belonged. While these students demonstrated mastery of all other attributes, they failed to understand the definition of ionization degree, apply its calculation formula, or analyze factors influencing ionization degree.

AMP22 (111011110) ranked third, comprising 9.75% of the students. This group could master all attributes except analyzing factors affecting ionization equilibrium and performing acid-base titration analysis.

*Learning Path*

The learning paths illustrating students' progression from no attributes (AMP1: 000000000) to full mastery (AMP27: 111111111) were mapped based on the inclusion relationships among AMPs (see Figure 3). For example, AMP4 (111000000) indicates that students have mastered C1, C2, and C3. AMP5 (111100000) shows the additional mastery of C4. AMP11 (111100100) demonstrates mastery of five attributes: C1, C2, C3, C4, and C7. From this, we can derive the path: (111000000) → (111100000) → (111100100).

**Figure 3**
*Learning Paths of Ionization Equilibrium*





https://doi.org/10.33225/jbse/25.24.742

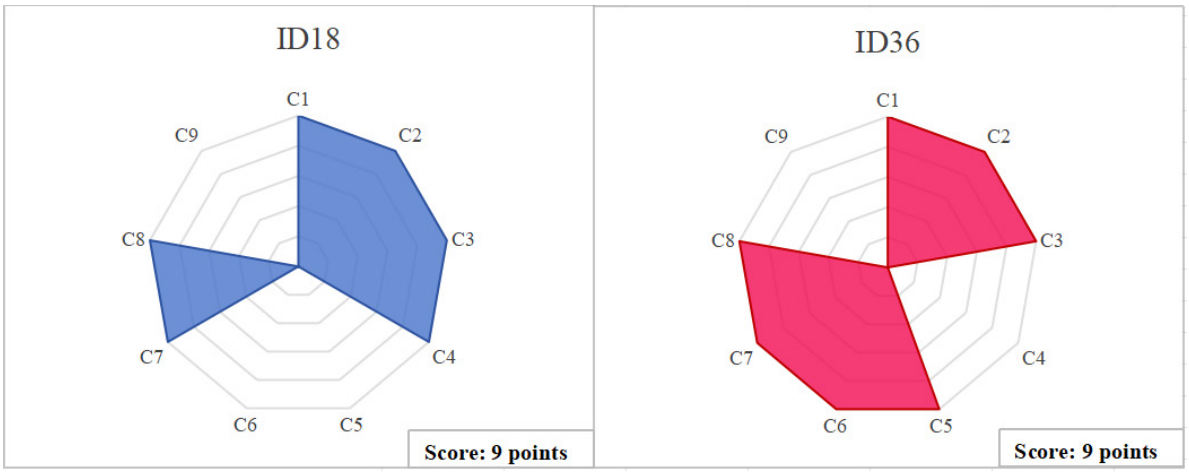
ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

As shown in Figure 2, the learning path with the sequence 000000000 ($\theta$: -3.78) → 100000000 ($\theta$: -3.34) → 110000000 ($\theta$: -2.87) → 111000000 ($\theta$: -2.44) →111010000 ($\theta$: -2.09) → 111010100 ($\theta$: -1.78)→ 111110100 ($\theta$: -0.90) → 111111100 ($\theta$: 0.61) → 111111110 ($\theta$: 2.38) → 111111111 ($\theta$: 3.57) contained the largest student cohort (N = 114). Notably, the monotonic increase in $\theta$-values of this path (from -3.78 to 3.57) reflects a consistent progression in cognitive ability levels. The learning sequence of this path followed the order: C1 (Electrolyte) → C2 (Strong/weak electrolyte) → C3 (Ionization equilibrium) → C5 (Ionization degree) → C7 (Ionic Product of Water) → C4 (The shift in ionization equilibrium) → C6 (Ionization equilibrium constant) → C8 (Solution acidity/alkalinity and pH) → C9 (Acid base titration).

*Personalized Cognitive Analysis*

In the personalized cognitive analysis, this study selected two students (ID18 and ID36) as research participants. As shown in Figure 4, both students scored 9 points on the test, but their attribute mastery patterns differed: (111100110) for ID18 and (111011110) for ID36, respectively.

**Figure 4**
*Comparative Analysis of Two Students' Mastery of Attributes with the Same Total Score*




Student ID18 had mastered C1 (Electrolyte), C2 (Strong/weak electrolyte), C3 (Ionization equilibrium), C4 (The shift in ionization equilibrium), C7 (Ionic Product of Water), and C8 (Solution acidity/alkalinity and pH), but needs to remediate C5 (Ionization degree), C6 (ionization equilibrium constant), and C9 (Acid-base titration). Student ID36 had mastered C1 (Electrolyte), C2 (Strong electrolyte/weak electrolyte), C3 (Ionization equilibrium), C5 (Ionization degree), C6 (Ionization equilibrium constant), C7 (Ionic Product of Water), and C8 (Solution acidity/alkalinity and pH), but needs to remediate C4 (The shift in ionization equilibrium) and C9 (Acid-base titration). The results of the students' personalized cognitive analysis show that students who achieve the same total score in the test may have different levels of understanding and mastery of the relevant knowledge.

## Discussions

*Students' Cognitive Strengths and Weaknesses in "Ionization Equilibrium"*

According to the students' attribute mastery probability, students had a good grasp of the attributes of "Electrolyte", "Strong /weak electrolyte", "Ionization equilibrium", "Ionic product of water", and "The shift in ionization equilibrium". This indicated that when students construct their understanding of "Ionization Equilibrium" topic, they could qualitatively understand the behavior of electrolytes in aqueous solutions. However, students did not


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

perform well in the attributes of "Ionization degree", "Ionization equilibrium constant", "Solution acidity/alkalinity and pH", and "Acid-base titration". This suggested that students had difficulties in quantitatively analyzing the ionization degree of electrolytes, the acidity and alkalinity of electrolyte solutions, and acid-base titration. These findings are similar to some existing studies. Lu et al. (2016) found that only a tiny number of students could explain the acid-base properties of solutions by quantitatively mastering the species and particular changes. Sheppard (2006) found that students had difficulty understanding what was happening to the pH values during a titration. However, different from those studies, the findings in this study provide more comprehensive and detailed diagnostic information. These findings systematically and concretely reveal which concepts make students feel difficult and hinder them from forming a complete knowledge structure in the concept network centered on ionization equilibrium.

*The Main Cognitive Structure Pattern of Students*

Applying the rule space model, this study identified 27 kinds of attribute mastery patterns among 318 students. According to the theory of cognitive diagnosis (Tatsuoka, 2009), each attribute mastery pattern represents a knowledge state of students. These knowledge states can reflect the characteristics of the cognitive structure formed by students after learning all the contents of "Ionization Equilibrium". The most common pattern (i.e., AMP15, 111110100) accounted for 11.01% of the students. These students could recognize electrolytes and non-electrolytes, strong electrolytes and weak electrolytes, qualitatively analyze the ionization equilibrium of weak electrolytes, understand self-ionization of water, and analyze the ionization degree of electrolytes. However, they could not calculate the ionization equilibrium constant, could not quantitatively analyze whether the solution is acidic or alkaline, and could not analyze acid-base titration. This is consistent with the students' learning situation reflected by the probability of mastering the attributes. It further proves that students have difficulties in quantitatively analyzing the acidity and alkalinity of electrolyte solutions and acid-base titration.

The 27 kinds of attribute mastery patterns among 318 students represent 27 kinds of knowledge states of students. This provides a reference for teachers to arrange group teaching or learning according to students' characteristics of cognitive structure. Diagnostic information of individual students obtained from personalized cognitive analysis may provide support for the arrangement of students' individualized learning.

*Primary Learning Path of Ionization Equilibrium*

On the basis of the theory of cognitive diagnosis, this study depicted the students' learning path according to their attribute mastery patterns. The learning sequence of the primary learning path was as follows: Electrolyte → Strong /weak electrolyte → Ionization equilibrium → Ionization degree → Ionic product of water → The shift in ionization equilibrium → Ionization equilibrium constant → Solution acidity/alkalinity and pH → Acid-base titration. The primary learning path may provide a potential developmental process for advancing students' understanding of "Ionization Equilibrium". It can provide reference for teachers to design teaching sequences and activities, bridging the gap between learners and the intellectual logic of the discipline.

## Conclusions and Implications

In conclusion, this study employed the Rule Space Model to systematically analyze the cognitive structures underlying students' understanding of the ionization equilibrium conceptual network, and thereby obtained an optimal learning path to construct this conceptual network. Based on the students' cognitive structure characteristics, teachers can strategically align instructional materials with these characteristics and implement targeted teaching strategies to address the cognitive weaknesses found. Furthermore, the proposed learning path can help teachers arrange structured instructional sequences to enhance conceptual understanding.

## Limitations of the Study

While this study yields valuable findings, several limitations warrant acknowledgment. First, the convenience sampling method employed to select participants from upper-secondary schools in Shaanxi Province, China,


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

resulted in a limited sample size, potentially constraining the generalizability of the results. Future studies should employ stratified random sampling across multiple regions to enhance external validity.

Second, the practical implementation of cognitive diagnostic outcomes remains underexplored. Subsequent research could implement quasi-experimental designs comparing intervention groups receiving learning path-aligned instruction with control groups following conventional pedagogy, thereby empirically validating instructional efficacy.

Third, the reliance on quantitative data neglects qualitative dimensions. Incorporating mixed methods through classroom observations and semi-structured student interviews would enable deeper exploration of latent cognitive processes and contextual learning behaviors.

**Acknowledgements**

This work was supported by the Excellent Graduate Training Program of Shaanxi Normal University (grant number LHRCTS23075).

**Declaration of Interest**

The authors declare no competing interest.

**References**

Alao, S., & Guthrie, J. T. (1999). Predicting conceptual understanding with cognitive and motivational variables. *The Journal of Educational Research*, *92*, 243–254. https://doi.org/10.1080/00220679909597602

Anderson, T. R. & Schönborn, K. J. (2008). Bridging the educational research-teaching practice gap: Conceptual understanding, part 1: The multifaceted nature of expert knowledge. *Biochemistry and Molecular Biology Education*, *36*(4), 309–315. https://doi.org/10.1002/bmb.20209

Artdej, R., Ratanaroutai, T., Coll, R. K., & Thongpanchang, T. (2010). Thai Grade 11 students' alternative conceptions for acid–base chemistry. *Research in Science & Technological Education*, *28*(2), 167–183. https://doi.org/10.1080/02635141003748382

Aydin-Gunbatar, S., & Akin, F. N. (2022). Pre-service chemistry teachers' use of pedagogical transformation competence to develop topic-specific pedagogical content knowledge for planning to teach acid–base equilibrium. *Chemistry Education Research and Practice, 23*, 137–158. https://doi.org/10.1039/D1RP00106J

Bransford, J. D., Brown, A. L., & Cocking, R. R. (2000). *How people learn: Brain, mind, experience, and school*. Washington. DC: National Academy Press.

Chen, F., Yan, Y., & Xin, T. (2016). Developing a learning progression for number sense based on the rule space model in China. *Educational Psychology*, *37*(2), 128–144. https://doi.org/10.1080/01443410.2016.1239817

Chen, F., Zhang, S., Guo, Y., & Xin, T. (2017). Applying the rule space model to develop a learning progression for thermochemistry. *Research in Science Education*, *47*, 1357–1378. https://doi.org/10.1007/s11165-016-9553-7

Cheong, I. P. A., Johari, M., Said, H., & Treagust, D. F. (2014). What do you know about alternative energy? Development and use of a diagnostic instrument for upper secondary school science. *International Journal of Science Education*, *37*(2), 210–236. https://doi.org/10.1080/09500693.2014.976295

Dogan, E., & Tatsuoka, K. (2008). An international comparison using a diagnostic testing model: Turkish students' profile of mathematical skills on TIMSS-R. *Educational Studies in Mathematics*, *68*, 263–272. https://doi.org/10.1007/s10649-007-9099-8

Field, A. (2009). *Discovering statistics using SPSS* (3rd ed.). London: Sage.

Gaigher, E., Rogan, J. M., & Braun, M. W. H. (2007). Exploring the development of conceptual understanding through structured problem-solving in physics. *International Journal of Science Education*, *29*(9), 1089–1110. https://doi.org/10.1080/09500690600930972

Gao, Y., Zhai, X., Cui, Y., Xin, T., & Bulut, O. (2022). Re-validating a learning progression of buoyancy for middle school students: A longitudinal study. *Research in Science Education*, *52*, 1761–1789. https://doi.org/10.1007/s11165-021-10021-x

Hadenfeldt, J. C., Bernholt, S., Liu, X., Neumann, K., & Parchmann, I. (2013). Using ordered multiple-choice items to assess students' understanding of the structure and composition of matter. *Journal of Chemical Education*, *90*(12), 1602–1608. https://doi.org/10.1021/ed3006192

He, P., Zheng, C., & Li, T. (2022). Upper secondary school students' conceptions of chemical equilibrium in aqueous solutions: Development and validation of a two-tier diagnostic instrument. *Journal of Baltic Science Education*, *21*(3), 428–444. https://doi.org/10.33225/jbse/22.21.428

Jiménez-Liso, M.R., López-Banet, L., & Dillon, J.(2020). Changing how we teach acid-base chemistry. *Science & Education*, *29*, 1291–1315. https://doi.org/10.1007/s11191-020-00142-6


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

Jin, H., & Wong, K. Y. (2023). Complementary measures of conceptual understanding: A case about triangle concepts. *Mathematics Education Research Journal*, *35*(1), 153–174. https://doi.org/10.1007/s13394-021-00381-y

Kousathana, M., Demerouti, M., & Tsaparlis, G. (2005). Instructional misconceptions in acid-base equilibria: An analysis from a history and philosophy of science perspective. *Science & Education*, *14*, 173–193. https://doi.org/10.1007/s11191-005-5719-9

Leighton, J. P., & Gierl, M. J. (2007). *Cognitive diagnostic assessment for education: Theory and applications*. Cambridge University Press.

Lin, C. T., Chang, S. J., & Chen, Y. H. (2022). Cognitive learning assessment based on fahp and rsm: A case study of introduction to network course. *Journal of Educational Computing Research*, *59*(8), 1543–1578. https://doi.org/10.1177/07356331211012685

Lu, S., & Bi, H. (2016). Development of a measurement instrument to assess students' electrolyte conceptual understanding. *Chemistry Education Research and Practice*, *17*, 1030–1040. https://doi.org/10.1039/C6RP00137H

McClary, L. M., & Bretz, S. L. (2012). Development and assessment of a diagnostic tool to identify organic chemistry students' alternative conceptions related to acid strength. *International Journal of Science Education*, *34*(15), 2317–2341. https://doi.org/10.1080/09500693.2012.684433

Milligan, A., & Wood, B. (2010). Conceptual understandings as transition points: making sense of a complex social world. *Journal of Curriculum Studies*, *42*(4), 487–501. https://doi.org/10.1080/00220270903494287

Ministry of Education, P. R. China. (2017). *Chemistry curriculum standards for senior high school*. People's Education Press.

Mintzes, J. J., Wandersee, J. H., & Novak, J. D. (2005). *Assessing science understanding: A human constructivist view*. Academic Press.

Nakiboglu, C., Rahayu, S., Nakibolu, N., & Treagust, D. F. (2024). Exploring senior high-school students' understanding of electrochemical concepts: patterns of thinking across Turkish and Indonesian contexts. *Chemistry Education Research and Practice*, *25*(1), 42–61. https://doi.org/10.1039/D3RP00124E

Özmen, H. (2008). Determination of students' alternative conceptions about chemical equilibrium: A review of research and the case of Turkey. *Chemistry Education Research and Practice*, *9*, 225–233. https://doi.org/10.1039/B812411F

Perkins, D. N. (1993). Teaching for understanding. *American Educator*, *17*, 28–35. https://doi.org/10.1080/00219266.1968.9653514

Planinic, M., Boone, W. J., Susac, A., & Ivanjek, L. (2019). Rasch analysis in physics education research: Why measurement matters. *Physical Review Physics Education Research*, *15*, 020111. https://doi.org/10.1103/PhysRevPhysEducRes.15.020111

Sheppard, K. (2006). High school students' understanding of titrations and related acid-base phenomena. *Chemistry Education Research and Practice*, *7*(1), 32–45. https://doi.org/10.1039/B5RP90014J

Stevens, S. Y., Delgado, C., & Krajcik, J. S. (2010). Developing a hypothetical multi-dimensional learning progression for the nature of matter. *Journal of Research in Science Teaching*, *47*(6), 687–715. https://doi.org/10.1002/tea.20324

Tatsuoka, K. K. (1983). Rule space: An approach for dealing with misconceptions based on item response theory. *Journal of Educational Measurement*, *20*, 345–354. https://doi.org/10.1111/j.1745-3984.1983.tb00212.x

Tatsuoka, K. K. (1995). Architecture of knowledge structure and cognitive diagnosis: A statistical pattern recognition and classification approach. In: Nichols, P. D., Chipman S. F., & Brennan R. L. (Eds.). *Cognitively diagnostic assessment* (pp. 327–361). Erlbaum.

Tatsuoka, K. K. (2009). *Cognitive assessment: An introduction to the rule space method*. London: Routledge.

Treagust, D. F. (1988). Development and use of diagnostic tests to evaluate students' misconceptions in science. *International Journal of Science Education*, *10*(2), 159–169. https://doi.org/10.1080/0950069880100204

Wu, X., Li, N., Wu, R.,& Liu, H. (2025). Cognitive analysis and path construction of Chinese students' mathematics cognitive process based on CDA. *Scientific Reports*, *15*(1), Article 4397. https://doi.org/10.1038/s41598-025-89000-5

Wu, X., Wu, R., Chang, H. H., Kong, Q., & Zhang, Y. (2020). International comparative study on PISA mathematics achievement test based on cognitive diagnostic models. *Frontiers in Psychology*, *11*, 2230. https://doi.org/10.3389/fpsyg.2020.02230

Wu, X., Wu, R., Zhang, Y., Arthur, D., & Chang, H. H. (2021). Research on construction method of learning paths and learning progressions based on cognitive diagnosis assessment. *Assessment in Education: Principles, Policy & Practice*, *28*(5–6), 657–675. https://doi.org/10.1080/0969594X.2021.1978387

Xin, T., Le, M. L., Guo, Y. F., & Jinag, Y. (2015). The approach to establishing achievement standard: The learning progressions based on cognition diagnostic. *Journal of Educational Studies*, *11*(5), 72–79. https://doi.org/10.14082/j.cnki.1673-1298.2015.05.009

Xin, T., Wang, C., Chen, P., & Liu, Y. (2022). Editorial: Cognitive diagnostic models: Methods for practical applications. *Frontiers in Psychology*, *13*, Article 895399. https://doi.org/10.3389/fpsyg.2022.895399

Yang, L., Huang, L., Wu, X., Xiong, J., Bao, L., & Xiao, Y. (2024). Assessment of preservice physics teachers' knowledge of student understanding of force and motion. *Physical Review Physics Education Research*, *20*(1), 010148. https://doi.org/10.1103/PhysRevPhysEducRes.20.010148

Yeo, J., Yang, H. H., & Cho, I. H. (2022). Using a three-tier multiple-choice diagnostic instrument toward alternative conceptions among lower-secondary school students in Taiwan: Taking ecosystems unit as an example. *Journal of Baltic Science Education*, *21*(1), 69–83. https://doi.org/10.33225/jbse/22.21.69

Zhan, P., Li, F., & Jiao, H. (2021). Editorial: Cognitive diagnostic assessment for learning. *Frontiers in Psychology*, *12*, Article 806636. https://doi.org/10.3389/fpsyg.2021.806636

Received: *April 16, 2025* Revised: *May 20, 2025* Accepted: *July 08, 2025*


https://doi.org/10.33225/jbse/25.24.742

ISSN 1648–3898 /Print/
ISSN 2538–7138 /Online/

Cite as: Wang, L., Ji, L., Zhang, H., Li, Y., & Zhou, Q. (2025). Cognitive diagnosis of upper-secondary students' conceptions in ionization equilibrium based on the rule space model. *Journal of Baltic Science Education*, *24*(4), 742–755. https://doi.org/10.33225/jbse/25.24.742

***Limin Wang*** PhD of Pedagogy Student, School of Chemistry & Chemical Engineering, Shaanxi Normal University, No.620, West Chang' an Road, Chang' an District, Xi'an City, Shaanxi Province, P. R. China
E-mail: wlm@snnu.edu.cn
ORCID: https://orcid.org/0009-0003-9818-5145

***Le Ji*** High School Chemistry Instructor, Xi' an Tieyi High School, No.120, Youyi East Road, Beilin District, Xi'an City, Shaanxi Province, P. R. China.
E-mail: 502312674@qq.com
ORCID: https://orcid.org/0009-0003-9613-0415

***Huiying Zhang*** PhD, Professor, School of Chemistry and Environmental Engineering, Hanshan Normal University, Dongshan Road, Xiangqiao District, Chaozhou City, Guangdong Province, P. R. China.
E-mail: zhy5158@126.com
ORCID: https://orcid.org/0009-0005-6244-8702

***Ye Li*** Associate Senior Editor, Shaanxi People's Education Press, No.58, Zhangba 5th Road, Yanta District, Xi'an City, Shaanxi Province, P. R. China.
E-mail: 16598727@qq.com
ORCID: https://orcid.org/0009-0009-0112-6901

***Qing Zhou*** *(Corresponding author)* PhD, Professor, School of Chemistry & Chemical Engineering, Shaanxi Normal University, No.620, West Chang' an Road, Chang' an District, Xi'an City, Shaanxi Province, P. R. China.
E-mail: Zhouq@snnu.edu.cn
ORCID: https://orcid.org/0000-0002-7071-7096